Eur. Phys. J. D **37**, 129–133 (2006)
DOI: 10.1140/epjd/e2005-00248-8

THE EUROPEAN PHYSICAL JOURNAL D

Lasing without inversion in an open Λ-type system: self-pulse and continuous wave

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Received 9 May 2005
Published online 6 September 2005 –

Abstract. Using the numerical calculation results, the transient evolution processes and steady outputs of the continuous wave lasing without inversion (LWI) originating from Pitchfork bifurcation instability and self-pulsing LWI originating from Hopf bifurcation instability in a resonant open Λ-type three level atomic system are studied. We found that the two kinds of LWI have some obvious differences not only from the steady outputs but also from the transient evolution processes. The effects of the unsaturated gain coefficient, cavity loss coefficient, ratio of the atomic injection rates and atomic exit rate on the transient evolution processes and steady outputs are discussed.

PACS. 42.50.Gy Effects of atomic coherence on propagation, absorption, and amplification of light; electromagnetically induced transparency and absorption – 42.50.Hz Strong-field excitation of optical transitions in quantum systems; multiphoton processes; dynamic Stark shift

1 Introduction

Since the seminal works of Kocharovskaya, Harris and Scully et al., lasing without inversion (LWI) has attracted tremendous attention (for example, see recent review [1] and papers [2–10] and references therein), because it can achieve laser actions in the short wavelength (e.g. X- even γ-ray) regions where lasing is impractical with conventional pumping schemes, and it has interesting statistical properties, such as narrower intrinsic line widths due to reduced spontaneous emission noises, which could be advantageous in, for example, optical communication networks. The various schemes have been put forward over the past few decades to realize LWI. The open V-type system [11,12] is one of the many schemes. Mompart et al. [13] pointed out that in a closed resonant Λ-type system the destabilization of the nonlasing solution appears only through the Pitchfork bifurcation. We found [14] that in an open resonant Λ-type system the destabilization of the nonlasing solution can be obtained not only through the Pitchfork bifurcation but also through the Hopf bifurcation, moreover we discussed the effects of parameters of the system on both bifurcations. From the nonlinear dynamics viewpoint, the lasing arising always corresponds to a loss of stability of the nonlasing stationary solution. Basing on the study of reference [14] now we explore the transient evolution processes and steady outputs of the continuous wave LWI originating from Pitchfork bifurcation instability and self-pulsing LWI originating from Hopf bifurcation instability, and discuss the effects of the gain parameter, damping rate, ratio of the atomic injection rates and atomic exit rate on the transient evolution processes and steady outputs of the two kinds of LWI.

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2 Basic equations

We consider the open resonant Λ-type system (Fig. 1). The driving field with Rabi frequency Ω is resonant with the transition $|1\rangle \leftrightarrow |2\rangle$; the weak probe field with Rabi frequency α is resonant with the transition $|1\rangle \leftrightarrow |3\rangle$, and an incoherent pump field with a pumping rate R is applied between levels $|1\rangle$ and $|3\rangle$. The transition between $|2\rangle$ and $|3\rangle$ is forbidden. The resonance condition allows us to take α and Ω as real and to seek solution of the problem such that the (slowly varying) density matrix element amplitudes of the medium in the interacting picture have the form

$$\rho_{12} = iy_{12}, \quad \rho_{13} = iy_{13}, \quad \rho_{23} = x_{23}, \tag{1}$$

where y_{12}, y_{13} and x_{23} are real time-dependent variables. In our notation, if $y_{13} < 0$, the system exhibits gain for the probe field; if $y_{13} > 0$, the probe field is attenuated. If $\rho_{11} - \rho_{33} < 0$ and $y_{13} < 0$, the LWI can be possible.

In the rotating-wave, slowing varying envelope and mean-field approximations, the system is governed by the

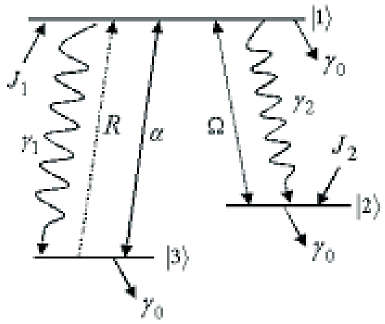

Fig. 1. An open Λ-type system.

set of Maxwell-Bloch equations

$$\begin{aligned}
d\rho_{11}/dt &= -(\gamma_1+\gamma_2+\gamma_0)\rho_{11}+R\rho_{33}+2\Omega y_{12}+2\alpha y_{13}+J_1\\
d\rho_{22}/dt &= \gamma_2\rho_{11}-\gamma_0\rho_{22}-2\Omega y_{12}+J_2\\
d\rho_{33}/dt &= \gamma_1\rho_{11}-(\gamma_0+R)\rho_{33}-2\alpha y_{13}\\
dy_{12}/dt &= -\Gamma_{12}y_{12}-\Omega(\rho_{11}-\rho_{22})+\alpha x_{23}\\
dy_{13}/dt &= -\Gamma_{13}y_{13}-\alpha(\rho_{11}-\rho_{33})+\Omega x_{23}\\
dx_{23}/dt &= -\Gamma_{23}x_{23}-\alpha y_{12}-\Omega y_{13}\\
d\alpha/dt &= -\kappa\alpha-gy_{13}.
\end{aligned} \tag{2}$$

In equations (2) the closure relation $\rho_{33}=1-\rho_{11}-\rho_{22}$ has been used; k and g denote the cavity loss coefficient and unsaturated gain coefficient, respectively; $\gamma_1(\gamma_2)$ the decay rate from level $|1\rangle$ to level $|3\rangle$ ($|2\rangle$); J_1 (J_2) and r_0 the atomic injection rate for level $|1\rangle$ ($|2\rangle$) and the atomic exit rate from the cavity, respectively. In the following discussion, we let $J_1+J_2=r_0$ for keeping the total number of the atoms as a constant and designate S $(=J_1/J_2)$. Γ_{12}, Γ_{13} and Γ_{23} are the atomic polarization damping rates, and in the radiative limit they can be expressed as

$$\Gamma_{12}=(\gamma_1+\gamma_2+R)/2,\ \Gamma_{13}=(\gamma_1+\gamma_2+2R)/2,\ \Gamma_{23}=R/2. \tag{3}$$

3 Transient evolution process and steady output of LWI

3.1 Continuous wave LWI

In reference [14], we have discussed the instability regions of the Pitchfork bifurcation in the resonant open Λ-type system. Now we explore the transient evolution process and steady output of the continuous wave LWI originating from Pitchfork bifurcation instability (statistic instability). Using the numerical calculation result from equations (2), Figures 2a–2d illustrate time evolution of Rabi frequency α of the continuous wave laser field (i.e. the probe field) for different parameters values, respectively. In the numerical calculation we take the initial condition as $\alpha=1\times10^{-10}$, $\rho_{11}=\rho_{22}=0$, $\rho_{33}=1$, other $\rho_{ij}=0$, $(i,j=1,2,3)$, and other parameters values are selected according to the result given in reference [14]. The values of g are given in MHz^2, and the other parameters values are given in MHz. Because that $\alpha=\mu_{13}E/\hbar$, and μ_{13} is the electronic dipole between atomic states $|1\rangle$ and $|3\rangle$, E is the amplitude of the probe field, so the time evolution of the probe field is same as that of α. Figure 2f is an amplification of the transient evolution at the beginning time corresponding to the case of Figure 2a. For the cases of Figures 2b–2d, the time evolution at the beginning time is similar to that shown by Figure 2f. By the analysis of the numerical result for different time intervals, we found that in the transient evolution process of the lasing field there are some smaller oscillations only at the beginning time as shown in Figure 2f and in the remainder time the oscillation of the lasing field doesn't occur.

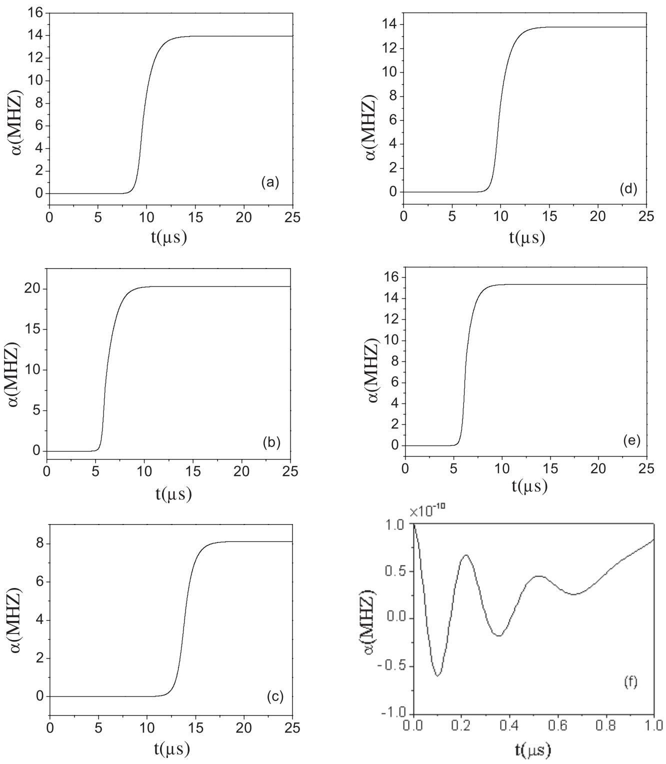

Fig. 2. Time evolution of continuous-wave laser field with $\gamma_{31}=0.8$, $\gamma_{21}=5.8$, $\Omega=10$, $R=2$. (a) $\kappa=1.0$, $g=1000$, $\gamma_0=0.2$, $S=3$; (b) $\kappa=1.0$, $g=3000$, $\gamma_0=0.2$, $S=3$; (c) $\kappa=3$, $g=1000$, $\gamma_0=0.2$, $S=3$; (d) $\kappa=1.0$, $g=1000$, $\gamma_0=0.2$, $S=15$; (e) $\kappa=1.0$, $g=1000$, $\gamma_0=0.8$, $S=3$.

From Figure 2 we can see that, the lasing field always reaches to a steady value, i.e. steady output, after going through two transient evolution stages, the first stage takes longer time interval and in this stage the lasing field is nearly unvarying, the second stage takes very short time interval but in this stage the lasing field is rapidly amplified, this amplification of the lasing field is result of the quantum coherent and interference. The transient evolution time before reaching to the steady situation and the steady output of the lasing field vary with the parameters values varying. Comparing Figure 2a with Figures 2b and 2c, we know that the effects of the cavity loss coefficient k and unsaturated gain coefficient g on the transient evolution time and steady output of the lasing field are just opposite; the bigger g is, the shorter the transient evolution time is and the larger the steady output is; the bigger k is, the longer the transient evolution time is and the smaller the steady output is. This is easy to understand from the physical means of g and k. Comparing Figure 2a with Figures 2d and 2e, we can see that, with the ratio of the atomic injection rates s $(= J_2/J_1)$ decreasing, the steady output of the lasing field increases and the evolution time needed for reaching to the steady output decreases; the atomic exit rate γ_0 decreasing will leads to a smaller steady output and a longer evolution time; but the numerical calculation result shows that when $\gamma_0 = 0$ (also $J_2 = J_1 = 0$, due to $J_2 + J_1 = s$), i.e. the closed system case, the steady output of the lasing field still exists. And this is consist with the conclusion given by Mompart et al. [13].

Figures 3a–3e show the transient evolution processes of the populations corresponding to Figures 2a–2e, respectively. Figure 3f is an amplification of the evolution process at the beginning time in Figure 3a, the evolution processes at the beginning time in Figures 3b–3e is similar to Figure 3f. From Figure 3 we can see that, there are three transient evolution stages before the populations reach to steady situations. The first stage takes shorter time, in this stage the populations oscillations occur as shown by Figure 3f. The second stage takes longer time, in this stage the populations don't vary. The third stage takes shorter time, and this stage corresponds to the second stage of the transient evolution process of the lasing field. In this stage the populations have obvious variations: the value of ρ_{11} decreases evidently while the value of ρ_{22} increases evidently; the value of ρ_{33} may increases or decreases and this depends on the parameter values. In the all cases of Figure 3, $\rho_{11} - \rho_{33} < 0$ is always satisfied, so the continuous lasing obtained in Figure 2 is LWI.

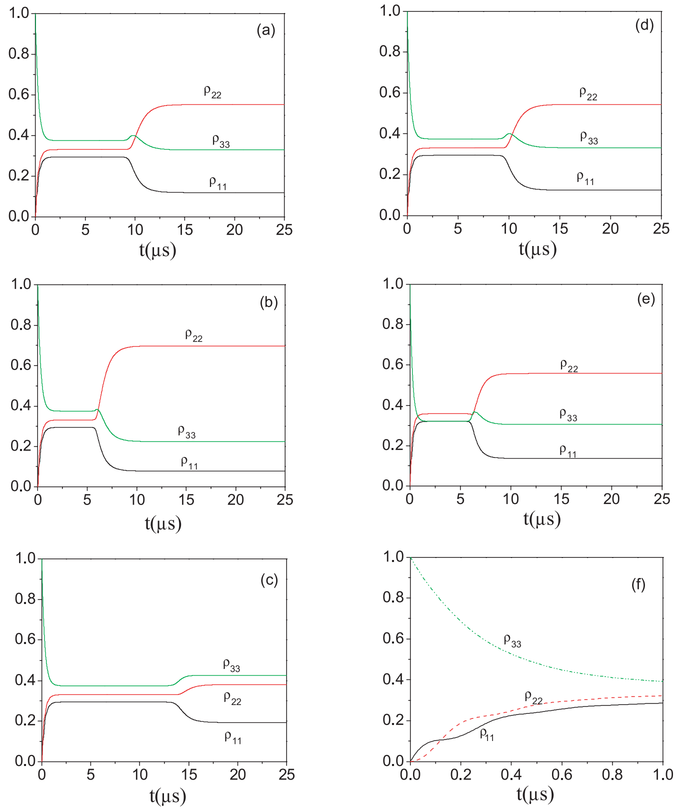

Fig. 3. Time evolution of the populations with the same parameters values as those in Figure 2.

3.2 Self-pulsing LWI

In reference [14], we have discussed the instability regions of Hopf bifurcation in the resonant open Λ-type system. Now we analyze the transient evolution process and steady output of the self-pulsing LWI originating from Hopf bifurcation instability (self-pulsing instability). Figure 4 illustrates the case where the initial condition is taken as $\alpha = 1 \times 10^{-10}$, $\rho_{11} = \rho_{22} = 0$, $\rho_{33} = 1$, and the other $\rho_{ij} = 0$ $(i, j = 1, 2, 3)$, and the other parameters values are selected as $\gamma_{31} = 5.8$, $\gamma_{21} = 0.8$, $\Omega = 10$, $R = 0.5$, $\kappa = 0.05$, $g = 10\,000$, $\gamma_0 = 4.8$, $S = 9$ according to the result given in reference [14]. The values of g are given in MHz^2, and the other parameters values are given in MHz. Figure 4a gives the complete picture of the transient evolution process and steady output of the lasing field while Figures 4b–4f present the details in some short times of the different evolution stages, respectively. From the numerical calculation result we know that, during the time interval from $t = 0$ to $t_1 \approx 15$ μs, the lasing field oscillates with a small increasing amplitude and the relative speed of the amplitude increasing is very fast, as shown in Figures 4b and 4c; in the time interval from $t_1 \approx 15$ μs to

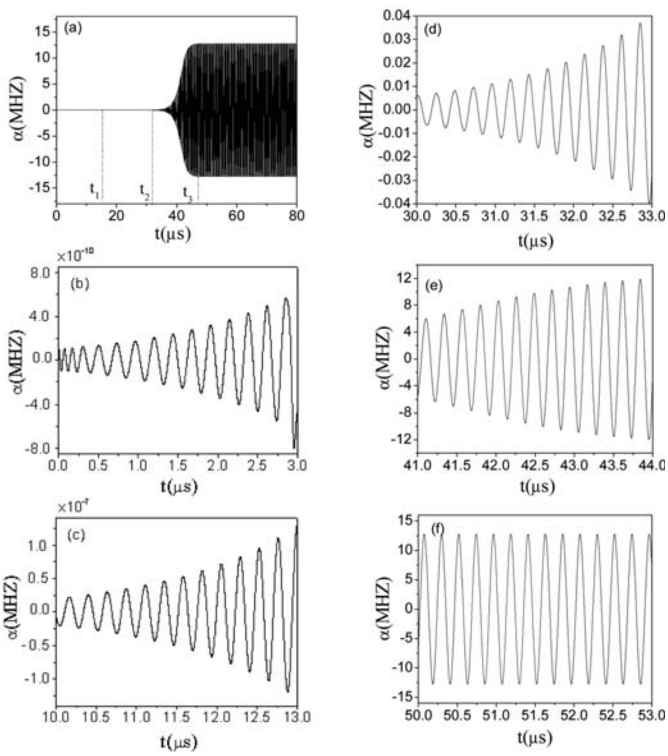

Fig. 4. Time evolution of self-pulsing laser field with $\gamma_{31} = 5.8$, $\gamma_{21} = 0.8$, $\Omega = 10$, $R = 0.5$, $\kappa = 0.05$, $g = 10\,000$, $\gamma_0 = 4.8$, $S = 9$.

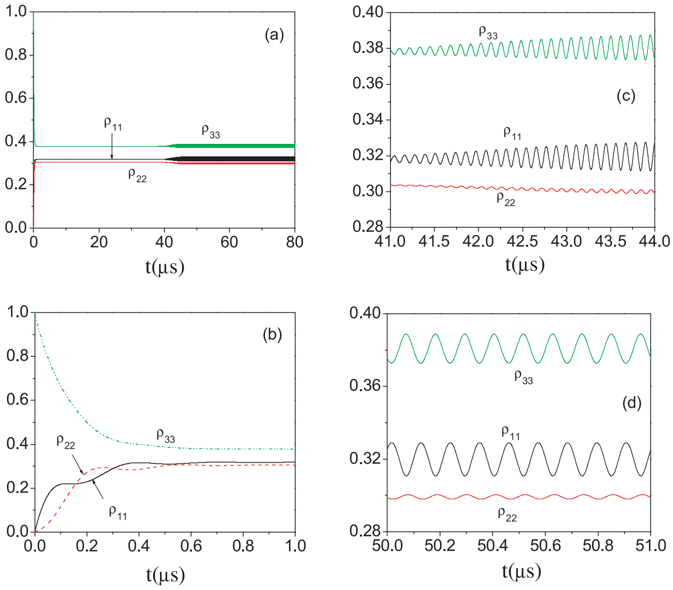

Fig. 5. Time evolution of the populations with the same parameters values as those in Figure 4.

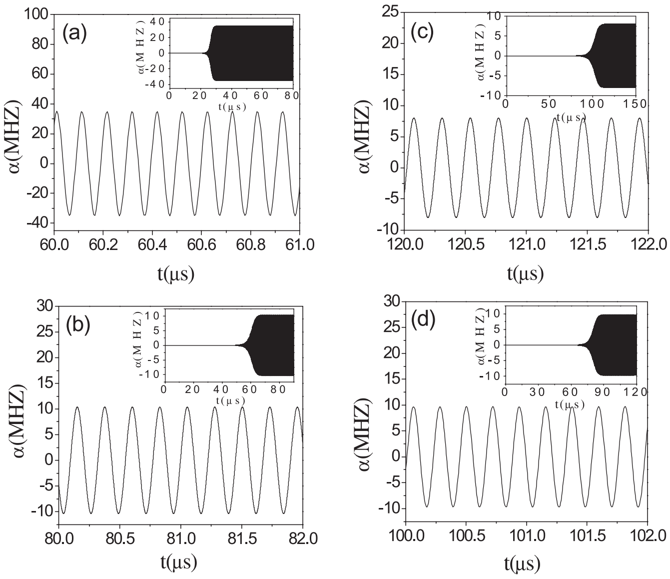

Fig. 6. Time evolution of self-pulsing laser field with $\gamma_{31} = 5.8$, $\gamma_{21} = 0.8$, $\Omega = 10$, $R = 0.5$ and (a) $\kappa = 0.05$, $g = 50\,000$, $\gamma_0 = 4.8$, $S = 9$; (b) $\kappa = 0.5$, $g = 10\,000$, $\gamma_0 = 4.8$, $S = 9$; (c) $\kappa = 0.05$, $g = 10\,000$, $\gamma_0 = 4.8$, $S = 7$; (d) $\kappa = 0.05$, $g = 10\,000$, $\gamma_0 = 4.7$, $S = 9$.

$t_2 \approx 32$ μs, the oscillation amplitude of the lasing field increases gradually with a smaller relative speed than before as illustrated in Figure 4d; and in the time interval from $t_2 \approx 32$ μs to $t_3 \approx 47$ μs, the oscillation amplitude increases rapidly as shown in Figure 4e; when $t > 47$ μs, the lasing field oscillates with a constant amplitude and a constant frequency, i.e. we get a steady output, as drawn in Figure 4f. In the transient evolution process before reaching to the steady output, the oscillation frequency of the lasing field has some small variation. Figure 4 shows that, due to interacting with the driving field and atomic system, the lasing field with the initial value equaling approximately to zero goes through the stages of the periodic oscillation with a very small but increasing slowly amplitude increasing slowly and periodic oscillation with increasing quickly amplitude, finally reaches to a steady self-pulsing output. It is very clear that the transient evolution process and steady output of the self-pulsing lasing originating from Hopf bifurcation instability is much different from that of the continuous wave lasing originating from Pitchfork bifurcation instability.

Figure 5 gives the transient evolution of the populations corresponding to Figure 4. The populations evolution is much different from that corresponding to the continuous wave lasing output except that both have some similarity at the beginning time (see Figs. 3f and 5b). Corresponding the steady output of the self-pulsing lasing field, the populations of the different levels aren't constant but oscillate with a same frequency and different amplitudes, this frequency equals approximately to two times of the oscillation frequency of the self-pulsing lasing field. The phase of ρ_{22} is same as that of ρ_{33} but just opposite to that of ρ_{11}. Since $\rho_{11} - \rho_{33} < 0$, so the self-pulsing lasing is LWI.

Using the same initial condition and the values of γ_{31}, γ_{21} and Ω but different other parameters values selected according to the result given in reference [14], We obtain the transient evolution process and steady output of the lasing field, which is similar to that shown by Figure 4. For simplicity, we only plot in Figure 6 the complete picture of the transient evolution process and the oscillation with constant amplitude corresponding to the steady output of the self-pulsing lasing.

Comparing Figure 4a with Figures 6a and 6b, we know that with $g(k)$ increasing, the oscillation amplitude of the steady self-pulsing output increases (decreases) evidently, and the evolution time needed for arriving at the steady oscillation decreases (increases) evidently. And comparing Figure 4a with Figures 6c and 6d, we find that with $\gamma_0(S)$ decreasing, the steady output decreases rapidly, and the evolution increases obviously. The numerical calculation result shows that when $r_0 = 0$ (also $J_2 = J_1 = 0$, due to $J_2 + J_1 = \gamma_0$), i.e. closed system case, the steady output of the lasing field doesn't exist. And this is consist with the conclusion given by Mompart et al. [13] that in a closed resonant Λ-type system the Hopf bifurcation, which leads to Self-pulse LWI, doesn't exist. In addition, k, r_0 and s varying has no effect on the oscillation frequency (period) of the lasing field as shown by Figures 4a, 6b, 6c and 6d, the oscillation frequency has the same value: $\omega = 2\pi/T \approx$ 28.2 MHz. The comparison of Figure 4a with Figure 6a tell us that when the value of$g$ increases obviously ($g =$ 50 000), the frequency also has a obvious decrement: $\omega' = 2\pi/T' \approx 62.8$ MHz.

The time evolution of the populations of the different levels corresponding to Figure 6 is similar to Figure 5. So, for simplicity we only give the time evolution corresponding to the steady self-pulsing lasing field as shown by Figure 7. From Figure 7 we can see that when the

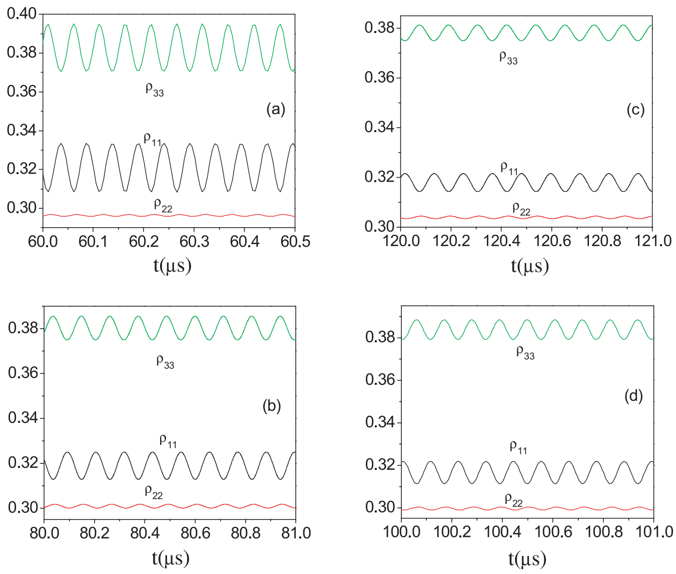

Fig. 7. Time evolution of the populations with the same parameters values as those in Figure 6.

lasing field oscillates, the populations of the different levels also oscillate with different amplitudes (the oscillation amplitude of ρ_{22} is very small); the oscillation frequencies of ρ_{11}, ρ_{22} and ρ_{33} are same, the phase of ρ_{22} is same as that of ρ_{33} but just opposite to that of ρ_{11}; for each case in Figure 7 there is always $\rho_{33} > \rho_{11}$, so the self-pulsing lasing obtained in Figure 6 is LWI.

4 Conclusion

In conclusion, basing on our previous study [14], by using the numerical calculation results from the density matrix equations of motion of an open resonant Λ-type system, the transient evolution processes and steady outputs for the continuous wave LWI and self-pulsing LWI are studied. The effects of the unsaturated gain coefficient, cavity loss coefficient, ratio of the atomic injection rates and atomic exit rate on the transient evolution process and steady output are discussed. We found that the transient evolution process and steady output of the continuous wave LWI are much different from those of the self-pulsing LWI. In the case of the continuous wave LWI, the lasing field doesn't oscillate except that at beginning time there is little oscillation with very small amplitudes. However, for self-pulsing LWI, the lasing field goes through the stages of the periodic oscillation with an increasing slowly very small amplitude and periodic oscillation with an increasing quickly amplitude, finally produces a steady output of self-pulsing LWI. In the two mechanism producing LWI, the effect of the unsaturated gain coefficient g and the cavity loss coefficient k on the lasing field is same: the amplitude of the lasing field increases with g increasing and decreases with k increasing, the time needed for reaching to steady output decreases with g increasing and increases with k increasing. In the two mechanism producing LWI, the atomic exit rate γ_0 decreasing always leads to a smaller steady output and a longer evolution time; but when $\gamma_0 = 0$, i.e. the corresponding closed system case, in the case of the continuous wave LWI, the steady output of the lasing field still exists while in the case of the self-pulsing LWI, the steady output of the lasing field disappears. And this is consist with the conclusion given by Mompart et al. [13] that in a closed resonant Λ-type system the Hopf bifurcation, which leads to self-pulse LWI, doesn't exist.

This work was supported by Key Laboratory for High Intensity Optics, Shanghai Institute of Optics and Fine Mechanics.

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